

Expeditious determination of the Bond Work Index (Wi) for ball milling

G. Viegas¹, F. Durão¹, C. Guimarães¹

¹, Instituto Superior Técnico, ULisboa, Lisboa, Portugal

September de 2020

Abstract

Dry grinding is one of the most important stages in several industries, such as chemical, mining and cement production, the energy requirements are high, this as motivated intensive research on this subject. One of the most widely employed empirical methodologies in the design of grinding equipment by engineers, particularly used in ball mills, was proposed by Fred C. Bond in the 1950's and is generically known as Bond method. This method is based on the application of the so-called Third Comminution Law (or Bond Law) and on the knowledge of one parameter, known as Work Index (Wi), which measures the ability to grind of a given mill/ material system.

The experimental determination of the Wi involves performing a standardized laboratory procedure that is very laborious, iterative and highly sensitive to the rigor of its execution.

It is proposed in this dissertation an alternative procedure, which is more expeditious and robust, using computational numerical simulation of the normalized Bond procedure, for the determination of the Wi for ball milling. The simulation is based on a calibrated kinetics mathematical model of laboratory batch ball milling and on an elementary model of laboratory sieving, considered perfect and ideal. The alternative procedure is compared with a case study and with the results of an equally expedite methodology proposed by Kapur.

Key words: Work Index; Bond Method; Ball Milling; Simulation

1. Introduction

Dry grinding is one of the most important stages in several industries, such as Chemical, Mining, and cement production, the energy requirements are very high. With a lot of manufacturers and a wide variety of grinding mills, the best fit selection is a difficult problem (Deniz et al., 2005).

In the past, ball mills were studied with a blackbox philosophy, where the interactions and interrelations of its elements were neglected, and the mill products were seen as a function of the feed. Such approaches led to the the development of the comminution laws of Kick, Rittinger and Bond (Tarasiewicz & Radziszewski, 1990).

When designing a grinding circuit, the Bond method is widely used to evaluate the performance and to determine mill power and size for a given material. This method is complex and time consuming. Furthermore, it is extremely sensitive to experimental errors. For this reason, many researches have proposed alternatives to the Bond method (Deniz, 2004; Deniz & Ozdag, 2003).

The alternative methods are based in kinetic and matricial models have been used in laboratory and in the industry. The kinetic model, is an alternative approach, which considers the comminution as a continuous process in which the particles rate of breakage of a given size interval is proportional to the mass of particles present in that same size interval (Umucu & Deniz, 2014).

Until today none of this new method was capable to replace the Bond Standard method, which continue to be one of the most used and reliable (Gharehgheshlagh, 2016). For that reason, in this work it is proposed and evaluated a simulation methodology of the standard Bond method, with the minimum laboratorial component possible that is replaced by simulation. That way we get reduced potential for experimental errors and, through a sensibility analysis on the simulation it is possible to calculate a confidence interval to Wi.

2. State of Art

In mining it was always a necessity to separate the mineral of interest from the waste rock. The

technological evolution enabled the extraction of poorer ores, increasing the existent reserves. To separate smaller particles the necessity for finer grinding arose, which led to the invention of crushers, granulators and rock mills (Balasubramanian, 2017).

Until the invention of the first rock mill, mid-nineteenth century, rocks were smashed using sledgehammers (Michaud, 2015). Now a days very different tools are used mainly in metal alloys, with diamond or wolfram drill bits, explosives and a wide range of machinery such as crushers and mills capable of reducing rock into powder. Of this machines one of the most versatile and used is the ball mill, however, despite being relatively cheap of acquiring and maintaining operational it is still very expensive, which leads to a carefully selected choice for the best fit (Pryor, 1965). Its cost made optimization a must to take the best possible advantage of its specifications shaping them to be a perfect match for each mine.

Through the observation of comminution processes, were developed the empirical laws of comminution. The main ones are known as the laws of Kick, Rittinger and Bond, also known as the first, second and third comminution laws, respectively (Austin et al., 1984).

The use of Kick and Rittinger theories has been met with a varied degree of success and are not realistic for the design of comminution circuits (Charles, 1957). On the other hand, Bond's Third Law can be applied to a range that ball and rod mills operate in. Despite the empirical basis of Bond's theory, it is a widely used method for designing ball and rod mills.

Charles, 1957, tries to adjust a pattern of classical grinding results to a distribution, using the distributions of Schumann and Rosin-Rammler. In both the distributions of probability was obtained a graphic with parallel lines which describe the evolution of each particle fraction size with time within the mill, what facilitates real live forecasts, based on case tests and laboratory trials. In this work the distribution used is the Rosin-Rammler because it is more used to describe particle size distributions in comminution processes.

Since the works of Brown (1941), Epstein (1948), Sedlatschek & Bass (1953), e Broadbent & Callcott (1956, 1957) the kinetic grinding equations started to be used more and more to describe the comminution process. Also known as the kinetic model, the batch grinding equations produced a huge amount of validated data to the point of Austin et al., 1984 concluded that there is no need to prove what makes this relations applicable, their the fruitful result of a reasonable hypothesis and in the form of the values of S (specific rate of

breakage) and B (primary progeny distribution). Being this true to describe an existing process, in a perspective of optimization this loses all its validity.

This work is focused on the Bond Work Index and its modelling, because it is considered the best method and therefor the more used in the mining industry, it has two major advantages on the others, being quite simple and applicable in most circumstances, not all tough. The original articles, written by Bond, were summarized in important publication in 1960, but a side from confuse had some mistakes, which led Rowland & Kjos (1978) to present this method and its practical application in a clear way. This latter publication is the one referenced to has the Bond method due to a greater and easier understanding of the method.

The modelling of the Bond laboratory test is made using grinding kinetics through a specific procedure to estimate the Bond grindability, which is a crucial factor. Other authors as Kapur (1970) and Karra (1981) try to simulate the Bond test to reduce the laboratory component due to its sensitiveness to errors and being so laborious.

Other authors tried to modulate this method and beased themselves on the first order kinetic grinding, some had more success than others, but as Bond said, after making one algorithm to facilitate this calculation, that this algorithms were not meant to replace the method, but to evaluate the day to day operations and to realise the necessary adjustments to the existing comminution installation, to optimize the installations to the heterogeneity of the rock masses and mineral deposits (Austin et al., 1984).

2.1 Kinetic model for laboratory batch grinding

The basic equations of the mathematical model for laboratory batch grinding, describe the mass variation of solid material in each class or granulometric interval i .

The mass balance equations describe the mass variation in each fraction i , M_{pi} , with time. The total mass of the material to be grounded, M , is constant, the mass balance equations can be simplified when dealing with the fractions mass/weight, p_i .

$$\frac{dp_i(t)}{dt} = -S_i P_i(t) + \sum_{j=1}^{i-1} b_{ij} S_j P_j(t), \quad (1)$$

$$n \geq i \geq j \geq 1$$

Equation 1 translates to, variation rate of material in fraction i equals to the destruction rate of particles in fraction i plus the rate of

particles generation in fraction i due to fragmentation of larger particles ($j=i+1$).

With n being the number of granulometric fractions, t the cumulative grinding time, $p_i(t)$ the mass fraction of the material in the i^{th} granulometric fraction, quantifying the fraction of material in the i^{th} granulometric fraction which is fragmented per unit of time, and $b_{i,j}$ is the function of fragmentation discretised, which describes the granulometric composition of the primary fragments generated by fragmentation of the material in fraction j now in fraction i .

Assuming parameter invariability S_i and $b_{i,j}$, $i,j=1,2,\dots,n$, with time t of grinding and $S_i \neq S_j$ for every para i,j , a solution to the linear ordinary differential equation system of constant coefficients as the following expression

$$p_i(t) = \sum_{j=1}^i d_{ij} p_j(0), n \geq i \geq 1 \quad (2)$$

where

$$d_{ij} = \begin{cases} 0, & i < j \\ e^{-S_i t}, & i = j \\ \sum_{k=j}^{i-1} c_{ik} c_{jk} (e^{-S_k t} - e^{-S_i t}), & i > j \end{cases} \quad (3)$$

com

$$c_{ij} = \begin{cases} -\sum_{k=i}^{j-1} c_{ik} c_{jk}, & i < j \\ 1, & i = j \\ \frac{1}{S_j - S_i} \sum_{k=j}^{i-1} S_k b_{ik} c_{kj}, & i > j \end{cases} \quad (3.1)$$

The laboratory kinetic batch grinding parameters, S_i and $b_{i,j}$, $i,j=1,2,\dots,n$, can be determined by experimental procedures described in detail by Austin et al., (1984), or determined by empirical mathematical relations validated with numerous experimental results. One relation largely used to describe the variation of S_i with the characteristic particle size (superior limit), ϕ , of each granulometric fraction i is given by Austin et al., (1984)

$$S_i = S_1 \left(\frac{\phi_i}{\phi_0} \right)^\alpha, \quad Q_i = \frac{1}{1 + \left(\frac{\phi_i}{\mu} \right)^\Lambda}, \quad n \geq i \geq 1 \quad (5)$$

with S_1 , α , μ and Λ parameters to adjust by minimizing the residues function between the granulometric distributions observer and simulated by the kinetic model.

The granulometric distributions of primary fragments, $b_{i,j}$, $i,j=1,2,\dots,n$, are usually defined from cumulative inferior distributions, $B_{i,j}^c$, as follows

$$\begin{cases} b_{i,j} = B_{i,j}^c - B_{i+1,j}^c, & 1 \leq j \leq i \leq n-1 \\ b_{n,j} = B_{n,j}^c \end{cases} \quad (6)$$

For the cumulative distributions, $B_{i,j}^c$, have been proposed some empirical relation such as the one proposed by Austin et al., (1984)

$$B_{i,j}^c = \begin{cases} 0, & \frac{\phi_{i-1}}{\phi_j} > 1 (i \leq j) \\ 1, & \frac{\phi_{i-1}}{\phi_j} > 1 (i \leq j), \\ \Phi_j \left(\frac{\phi_{i-1}}{\phi_j} \right)^\gamma + (1 - \Phi_j) \left(\frac{\phi_{i-1}}{\phi_j} \right)^\beta, & \frac{\phi_{i-1}}{\phi_j} \leq 1 (i > j) \end{cases} \quad (7)$$

com $\Phi_j = \Phi_0 \left(\frac{\phi_j}{\phi_0} \right)^{-\delta}, \phi_0 = 1 \text{ mm}$

with γ , β , Φ_0 and δ parameters to estimate, or the one proposed by R. P. King (King, 2001)

$$B_{i,j}^c = \begin{cases} 0, & \frac{\phi_{i-1}}{\phi_j} > 1 (i \leq j) \\ 1, & \frac{\phi_{i-1}}{\phi_j} > 1 (i \leq j), \\ \left(\frac{\frac{9}{\left(\frac{\phi_j}{\phi_{i-1}} \right)^{-1}}}{1 - (1 - t_{10})} \right)^\gamma, & \frac{\phi_{i-1}}{\phi_j} \leq 1 (i > j) \end{cases} \quad (8)$$

with t_{10} and γ parameters to estimate.

2.2 Bond Standard Method

The Bond Standard Method (Bond & Maxson, 1943) states that the Work index (Wi) is determined by achieving a circulating load of 250% in a dry batch grinding (Bond, 1949, 1952, 1961).

Usually it takes 7 to 10 cycle to obtain stationarity and using the mean grinding velocity of the last three cycles it is obtained the

Bond's grindability (G_B). The Work index is then calculated with the following equation.

$$W_i = 1,1 \frac{44.5}{MOG^{0.23} \times G_B^{0.82} \times \left(\frac{10}{\sqrt{x_P}} - \frac{10}{\sqrt{x_F}} \right)} \quad (9)$$

Where: W_i – Work index (kwh/t)
 MOG – Mesh of grid (μm)
 G_B – Bond grindability index (g/rev)
 x_F – Mesh of grid that allows passage to 80% of the mil feed (μm)
 x_P – Mesh of grid that allows passage to 80% of the mil product (μm)

When studying circuits that differ from the conditions pre-incorporated by Bond, Rowland, 1998 suggested a series of efficiency factors (EF) to correct the values of power required for the mills and the validation of the W_i determined in the laboratory. Kapur (1970) alternative method is used to compare both results the Bond experimental test and the Bond simulation and it states that the fact that exists a relation, precise and relatively simple, between G_2 and G_b it makes possible to calculate Bond's work index directly. This is achieved using the following equation

$$W_i = K[MOG]^a[-G_2]^b[rM_1]^c[1-r]^d \quad (10)$$

The unknown constants K and the exponents were calculated with the least square sum applied to a test which the results were known to the author. The Bond Work Index was calculated using the formula

$$W_i = 44.5 \left[MOG^{0.23} G_b^{0.82} \left[\frac{10}{\sqrt{x_P}} - \frac{10}{\sqrt{x_F}} \right] \right]^{-1} \quad (11)$$

The resulting statistical equation is

$$W_i = 2.648[MOG]^{0.406}[-G_2]^{-0.810} \times [rM_1]^{-0.853}[1-r]^{-0.099} \quad (12)$$

Both equations above give results in kwh/st and have to be multiplied by 1.1 to convert the results to kwh/t.

3. Methods and Materials

In order to validate the alternative method in analyse, a kinetic grinding test was performed, which the software uses to create a mathematical model and simulates a Bond test, a Bond test was also performed to validate the software result. This software aims to obtain a reliable simulation of a Bond test through a

kinetic grinding test, calculating the Bond work index (W_i) and its confidence interval without resorting to the Bond standard method itself, which is highly sensitive to experimental errors - which in turn are difficult to minimize, predict and quantify.

3.1 Samples preparation

According to the Bond test, the samples must have a particle size of less than 3550 μm and enough quantity for the required repetitions to stabilize the test. In this way, the sample, a limestone block, was reduced by means of a sledgehammer, two jaw crushers with different sizing gap settings and a roll mill. Since the roll mill was the last step, the sizing gap was adjusted to the maximum desired size of 3.55 mm. After grinding in the roll mill, the product was sieved at 3550 μm . The product infra, known as the product of roll mill infra 3550 μm (PMR -3550 μm), was stored and the supra screen returned to the roll mill until sufficient PMR -3550 μm material was obtained for the tests. Using a jones splitter, 9 samples were created with approximately 466.67g – four of them were used to describe the initial sample particle sizes, another four for the kinetic tests and one for a Bond test.

3.2 Screaming

The several sieved products had different masses by size intervals and as such, different additional cares. Dry sieving, usually for twenty minutes, was used for initial sample (AG's) from the size 2800 to 355 μm , for the ball mill products (PMB's) from the size 2800 to 500 μm , due to the quantity of finer particles dry sieving below 500 μm was inefficient. Below the sizes referred before was used wet sieving with a shaking table, usually thirty minutes, until de 45 μm sieve. Below 45 μm the material was retained in a bucket. When wet sieving with great quantities of fines the chance of clogging the sieves is huge, so every sample for wet sieving greater than 150g was divided in two to prevent this from happening. The sieving column was prepared maintaining a ratio between sieves of square root of two, and the sieves used and 200x50mm.

Between every screened samples, the sieves were carefully cleaned to minimize possible contamination and loss of material.

In the Bond Standard test each cycle ends with a dry sieving at 125 μm , this sieving was done with a single sieve on a shaking plate. In order to obtain the best possible screening, the material was sieved for 10 minutes, the product infra 125 μm was then weighted and stored,

only after two consecutive weights below one gram was this procedure stopped, until then the material supra 125 μm was sieved for periods of 10 minutes and weight. The final product of this test were divided between supra 125 and infra 125, both products were analysed according to the sieving method already explained the supra 125 μm was dry sieved from 2800 to 125 μm and the infra 125 μm was wet sieved from 125 to 45 μm .

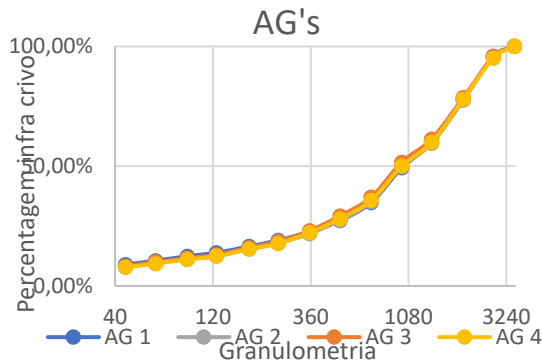


Figure 1 – Size distribution analyses of initial sample.

The grain size curves of all the AG's are almost identical, figure 1, and follow the same tendency. The average of loss material was 0.4134% and the major losses were seen in the PMB's, probably due to the inefficient cleaning of the ball mill and the ball and the presence of a bigger percentage of fine particles. The sampling error was considered vestigial.

The kinetic test was made, aiming to obtain the grain size distributions of four different milling times, so that the algorithm can calculate the kinetic parameters necessary to create the kinetic grinding model of our sample. The milling times were half a minute, one, two and four minutes. This times were chosen to best describe the behaviour of the sample material in the mill, the resulting products were called PMB 0.5 for the half minute PMB 1 for the one minute and so on.

The ball mill used was the Denver mill, due to its differences to the Bond mill, some adaptations had to be made which can be seen in the next tables.

Tabela 1 – Denver mill specs

D_m (cm)	30.48
L_m (cm)	12.7
V_{rot} , rpm	70
M_B , g	9060.51
Forro do moinho	Liso
Tipo de moagem	Seca
V_A , cm^3	291.67
M_A , g	466.67

After milling, the mil and ball were cleaned to prevent loss of material, the resulting material was weighted and stored for size analysis. Seen that for the Bond test it is required to obtain a circulating load of 250%, we want to obtain 133.33g of infra 125 μm from the 466.67g of initial load. According to the AG's size analysis the initial sample has 12,94% of infra 125 μm from the required 28,57%.

3.3 Bond Test

Firstly, was defined the 125 μm mesh of grid as the test sieve, then using an initial sample under the same grinding conditions used for the kinetic test, taking into account the grain size curves of the kinetic test, the initial time of this test was defined at two minutes to ensure that the optimal time would be less (i.e., it was done from the limits to the centre).

The method of introducing the sample and the balls into the mill was identical to the method used for the grinding kinetics test.

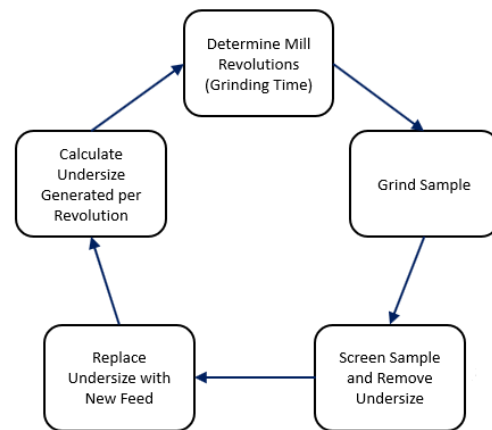


Figure 2 – Bond's locked grinding cycle scheme

The purpose of this test is to reach a circulating load of 250%, that is, 28.57% of material infra 125 μm after the completion of the test. With this aim, the product of each cycle was dry sieved on the test screen. Then, the infra product was removed, weighed and compared to the target value. To the supra 125 fraction new sample material was added to in sufficient quantity to redo approximately 466.67g. The next cycle starts after making the necessary adjustments to the grinding time, figure 2. It is important to note that the refills of sample material were also prepared with successive divisions in Jones splitter. When material the infra 125 μm was more than the objective, the grinding time was reduced (or the other way around). The addition of material between Bond cycles represents the infra product plus the losses inherent to the test. At the end of eleven cycles, stationarity was considered attained and, after dry sieving with the test screen, the Bond test was concluded.

Due to the randomness of the grinding behavior, it was extremely difficult to maintain a fixed time and obtain the desired circulating load. The test was finished without performing the three stable repetitions at the same time, once that in the fourth, fifth and ninth cycles that used the same grinding time the products were very different which means stabilization of the grinding time may not be possible for this test in a Denver mill. This was not such a problem because an average of the last three values would be made as recommended in most of the literature concerning this test. Throughout this test, the losses in the grinding were 0.654% and the losses in the screening were 0.113%.

3.4 Simulation

The modelling present in this work was made in two very similar excel workbooks being the only difference the algorithm behind the creation of the mathematical model, one using the formulas of Austin, Klimpel and Luckie (equation 7) and the other uses a newer version of this formulas revised and modified by R. P. King (equation 8). The workbook used is divided into two parts, firstly it requires the sample initial size distribution, and the size distributions of the products of the kinetics test, with this information it creates a mathematical model for the grinding kinetics of this sample in this mill, the second part consists in the simulation of the Bond test.

For the creation of the mathematical model, were tried three different objective functions Square Sum Residues, Sum of Absolute Relative Residues and Weighted Square Sum of Residues and for each of these functions three to four times of the kinetic test. The model selection is done through the next three worksheets of the excel which evaluate the graphical adjustment of the created model and the given data, the Qui-Square and the Kolmogorov-Smirnov tests.

3.5 Best Fit Selection

With the kinetic modeling test done, we can choose in the cell 'No. Of tests' the number of products of the kinetic test that we want to use to adjust the parameters of the kinetic model. Then it is possible to choose one of three functions for the objective function, and using two buttons (under the tables next to the numbers 2 and 3) two macros can be activated sequentially. The first one, marked by number 2, adjusts the kinetic parameters of a closed-circuit grinding using an excel tool called Solver. This adjustment starts as a non-linear restricted optimization problem, with a view to minimizing the selected objective function. Considering the

existing restrictions (already introduced into Solver), it runs as an iterative process and uses a generalized reduced nonlinear gradient function. This excel tool informs if an optimal solution was obtained or if the solution was obtained by convergence through two cells—one with the name 'Solver Info' followed by a number (0 if optimal and 1 if the solution was by convergence). The button below the table marked by the number 3 calculates the standard deviation of the estimated parameters. The excel also features a worksheet (model validation), two for residual analysis using the Chi-square test and the Kolmogorov-Smirnov test. After obtaining the best fit, it is possible to proceed to the page where the W_i is calculated through the Bond laboratory test and compared with the W_i calculated by the Kapur method. The next step is in the excel page 'Simulated Bond test' where, through the mathematical model chosen and validated previously, it is possible to simulate a test using formulas 7 or 8. For this simulation, it is necessary to fill the green cells in the excel page "Simulated Bond Test", with the due values converted to the Bond mill. Note that this page has no connection with the 'Experimental Bond Tests', both are independent. From the Excel user's point of view, it consists of pressing the 'start simulation' button to obtain a first cycle, and then pressing the 'continue simulation' button, which simulates one more cycle whenever it is pressed. The simulation should end when convergence is reached. There is a table containing the simulated values of x_F , x_P , and W_i . In parallel with the simulated data, the Kapur method is used as an alternative method for the simulated values, where, it is necessary to perform the residues minimization operation.

3.6 Sensitivity Analysis

In order to obtain statistical relevance of the calculated data, in this Excel exists a worksheet for the calculation of the trust interval through a sensibility analysis of the results. In table 2 it is possible to see the results obtained.

Table 2 - Results table and confidence interval

Teste de Bond Experimental		Teste de Bond Simulado	
W_i	7,39	W_i	8,25
$W_i - 2\sigma =$	6,34	$W_i - 2\sigma =$	6,89
$W_i + 2\sigma =$	8,45	$W_i + 2\sigma =$	9,61
$IC_{(95\%)} =$	[6,34: 8,45]	$IC_{(95\%)} =$	[6,89: 9,61]

4. Results and Discussion

The Kapur method was chosen as the alternative methodology in order to have a known and trustworthy method to serve as comparison. In both situations the value of the proposed alternative method is within a 95% confidence interval.

We also noticed that the experimental Bond test confidence interval comprises all the obtained values, which demonstrates its high probability of reproducing reliable and repeatable values. This is important to note due to the high sensitivity of this laboratory method to operator errors. In this work, it was possible to show that the program under analysis is in fact a valid option for the calculation of Bond's Work Index, showing, for the sample under study, that the EXCEL provides a good result. This EXCEL has clear advantages both in terms of quick calculation, avoiding a prolonged and careful Bond test, as well as with regard to the study of the error associated with the calculation—the error in this case is easier to calculate and more certain, because the experimental errors are not measurable and another form of validation is required through alternative methods or tests repetition. It was also verified that the AKL model for the studied sample results in a better mathematical model than the R.P.King model, despite the fact that the EXCEL can use both. Besides that, it was also possible to corroborate, comparing with the literature, that the fact that it was used a mill of smaller dimensions than the Bond mill, the expression of the error is clearly greater and consequently the stabilization of the test is more difficult and erratic.

5. References

Austin, L. G., Klimpel, R. R., & Luckie, P. T. (1984). *Process Engineering of Size Reduction - Ball Milling*. SME.

Bond, F. C. (1949). Standard grindability test tabulated. *Transactions of the American Institution of Mining Engineers*, 183, 313–329.

Bond, F. C. (1952). The Third Theory of Comminution. *AIME*, 193, 484–494.

Bond, F. C. (1960). Crushing and Grinding Calculations. *British Chemical Engineering*, 6, 378–391.

Bond, F. C., & Maxson, W. L. (1943). Standard grindability tests and calculations. *Transactions of the Society of Mining Engineers of AIME*, 153, 362–372.

Deniz, V. (2004). Investigation of breakage behaviour of two different pumice stones.

The European Journal of Material Processing and Environmental Protection, 4(2), 162–167.

Deniz, V., Akkurt, Y., & Umucu, Y. (2005). A New Model on Breakage Behaviour of A Laboratory Impact Mill. *Proceedings of the 19th International Mining Congress of Turkey, Izmir.*, 229–232.

Deniz, V., & Ozdag, H. (2003). A new approach to bond grindability and work index: Dynamic elastic parameters. *Minerals Engineering*, 16(3), 211–217. [https://doi.org/10.1016/S0892-6875\(02\)00318-7](https://doi.org/10.1016/S0892-6875(02)00318-7)

Gharehgheshlagh, H. H. (2016). Kinetic grinding test approach to estimate the ball mill work index. *Physicochemical Problems of Mineral Processing*, 52(1), 342–352. <https://doi.org/10.5277/ppmp160129>

Kapur, P. C. (1970). ANALYSIS OF THE BOND GRINDABILITY TEST. *Transactions of the Institution of Mining and Metallurgy, Section C: Mineral Processing and Extractive Metallurgy*, 79(763), 103–108.

King, R. P. (2001). *Modeling and Simulation of Mineral Processing Systems* (Elsevier (ed.); 1 st Editi). Butterworth-Heinemann.

Rowland, C. A. (1998). Using the Bond work index to measure operating comminution efficiency. *Minerals and Metallurgical Processing*, 15(4), 32–36. <https://doi.org/10.1007/bf03403155>

Tarasiewicz, S., & Radziszewski, P. (1990). : *Batch Grinding Equation and Breakage Energy [I. 25*.

Umucu, Y., & Deniz, V. (2014). The Evaluation of Grinding Behaviors of Quartz and Feldspar. *Tojsat*, 4(1), 60–67.